

A Reader's Companion

Mathematics, science, and philosophy for readers of
A Mathematical and Scientific Approach to the Unseen

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Companion to: *A Mathematical and Scientific Approach to the Unseen* (Book 1)

Edition: First edition

Contents

Before You Start	6
1 Sets and the Unseen	8
1.1 The idea	8
1.2 By hand	8
1.3 In Octave	9
1.4 In Python	9
1.5 Read further	9
2 Logic and What Follows from What	11
2.1 The idea	11
2.2 By hand	11
2.3 In Octave	12
2.4 In Python	12
2.5 Read further	12
3 Probability and Changing Your Mind	13
3.1 The idea	13
3.2 By hand	13
3.3 In Octave	14
3.4 In Python	14
3.5 Read further	15
4 Rules That Update Step by Step	16
4.1 The idea	16
4.2 By hand	16
4.3 In Octave	16
4.4 In Python	17
4.5 Read further	17
5 Lists of Answers, Differences, and Limits	18
5.1 The idea	18
5.2 By hand	18
5.3 In Octave	19
5.4 In Python	19
5.5 Read further	20
6 Comparing Priorities as Shares	21
6.1 The idea	21
6.2 By hand	21
6.3 In Octave	21
6.4 In Python	22
6.5 Read further	22

7	Sharing Credit Fairly	23
7.1	The idea	23
7.2	By hand	23
7.3	In Octave	24
7.4	In Python	24
7.5	Read further	25
8	Choices That Depend on Other People’s Choices	26
8.1	The idea	26
8.2	By hand	26
8.3	In Octave	26
8.4	In Python	27
8.5	Read further	27
9	Systems, Feedback, and Chaos	28
9.1	The idea	28
9.2	By hand	28
	9.2.1 Feedback	28
	9.2.2 Chaos	29
9.3	In Octave	29
9.4	In Python	29
9.5	Read further	30
10	Waves, Frequency, and the Spectrum	31
10.1	The idea	31
10.2	By hand	31
10.3	In Octave	32
10.4	In Python	32
10.5	Read further	32
11	The Physics the Book Refers To	33
11.1	The early universe	33
11.2	Quantum entanglement	33
11.3	The exclusion principle and surfaces	34
11.4	Thermodynamics, entropy, and thresholds	34
12	Chains, Trees, and Networks	35
12.1	The idea	35
12.2	By hand	35
12.3	In Python	35
12.4	Read further	36
13	A Philosophy Toolkit	37
13.1	Kinds of reasoning	37
13.2	Knowledge and its sources	37
13.3	Testing a claim	37
13.4	Correlation and cause	37
13.5	Analogy	38
13.6	Models	38
13.7	The Stoic division	38

13.8	The limits of formal proof	38
13.9	Read further	39
14	Finding Your Way	40
14.1	Which part goes with which chapter	40
14.2	Symbols used in the main book	41
14.3	Where to learn more	41

Before You Start

You are reading the main book and you have reached a line of symbols. Perhaps you last saw a formula at school, or perhaps you never had the chance to study this kind of material at all. This booklet is for you. It takes each piece of mathematics, science, and philosophy the book uses and explains what it means, how to work a small example yourself, and where to learn more.

You can stop reading the main book at any formula, come here, work the example, and go back. Nothing in the main book depends on your being able to do the calculation. Being able to do it simply means you can test the reasoning for yourself, and that is worth having.

What the mathematics is for

The main book stands on the Quran and the reliably transmitted Sunnah. Its mathematics and science serve a smaller purpose. A formula sets out a line of reasoning so that every step can be inspected. A physical comparison makes an unfamiliar idea easier to picture. If one of those models turns out to be wrong, the model is corrected or dropped, and revelation is untouched.

The book marks its claims with three labels, and this booklet keeps them. [A] means an established source or finding. [B] means an explanatory model or comparison. [C] means a hypothesis nobody has verified. Almost everything you will calculate here is a [B]. The arithmetic will be exact, and the question of whether the model fits the world will still need its own support. Each part ends with a short orange box that says what the calculation leaves open.

Three ways to work

Every example is worked three times, and you can use whichever suits you.

The first way is paper, a pencil, and an ordinary calculator. A phone calculator is enough. Work the hand version first if you can, because writing the steps out is how the idea settles.

The second way is GNU Octave, a free program for numerical work. You type a line, press Enter, and it answers. Octave leaves the semicolon off a line to mean “print the result”, which makes it a good first tool. Install it from octave.org.

The third way is Python, a free general-purpose programming language. It takes a few more words to say the same thing, and it is the language you are most likely to meet elsewhere. Install it from python.org. The examples need nothing beyond the standard installation.

Every listing in this booklet is also a file you can download from the book’s website and run unchanged. The file name appears in the text above each listing.

Reading a formula

When you meet a formula, read it the way the main book suggests. First find the question it answers. Next ask what each symbol stands for. Then follow the calculation with small numbers of your own.

A table of the symbols used in the main book is at the end of this booklet, in Part 14.

Sets and the Unseen

1.1 The idea

A *set* is a collection of things, and the things are its *members*. The set of days in a week has seven members. We write a set by listing its members inside braces: $\{1, 2, 3\}$.

Four operations do most of the work.

Symbol	Meaning
$x \in S$	x is a member of S .
$S \subseteq T$	S is a <i>subset</i> of T . Every member of S is also in T .
$S \cup T$	The <i>union</i> . Everything that is in S , in T , or in both.
$S \setminus T$	The <i>difference</i> . Everything in S that is absent from T .

Where the book uses this

The chapter *A Precise Definition of 'The Unseen'* writes $U(t) = \Omega \setminus M(t)$. Here Ω is everything that actually occurs in created reality, $M(t)$ is what has been established by stage t , and $U(t)$ is what remains unavailable at that stage. The chapter then splits $U(t)$ into the absolute unseen G (*Ghayb Mutlaq*) and the relative unseen $N(t)$ (*Ghayb Nisbi*). The same model returns in *The Beginning and the Limits of Physics* and *When Rejected Ideas Later Enter Science*.

1.2 By hand

Shrink the model until it fits on a page. Let Ω hold ten things, numbered 1 to 10. Suppose creatures could in principle come to establish items 1 to 7, so $A = \{1, \dots, 7\}$. The rest, $G = \Omega \setminus A = \{8, 9, 10\}$, is beyond them.

By stage 1, people have established $M(1) = \{1, 2\}$. Cross those out of Ω and you have what is unavailable:

$$U(1) = \Omega \setminus M(1) = \{3, 4, 5, 6, 7, 8, 9, 10\}.$$

Cross them out of A instead and you have the relative unseen:

$$N(1) = A \setminus M(1) = \{3, 4, 5, 6, 7\}.$$

Test the book's first equation, $U(t) = G \cup N(t)$, by joining $\{8, 9, 10\}$ to $\{3, 4, 5, 6, 7\}$. You get $U(1)$ back.

By stage 2, two more items are established, so $M(2) = \{1, 2, 3, 5\}$. Now $U(2) = \{4, 6, 7, 8, 9, 10\}$. Two things have happened, and the book writes each as a line of symbols. Knowledge grew, $M(1) \subseteq M(2)$. The unseen shrank, $U(2) \subseteq U(1)$. Notice what did not move: items 8, 9, and 10 are in U at every stage.

1.3 In Octave

The file is `sets_unseen.m`.

```
% Sets and the Unseen model:  $U(t) = \Omega \setminus M(t)$ .
omega = 1:10;           % every state that actually occurs
A = 1:7;                % states creatures could in principle establish
G = setdiff(omega, A);   % absolute unseen
M1 = [1 2];             % established by stage 1
M2 = [1 2 3 5];         % established by stage 2

U1 = setdiff(omega, M1)
N1 = setdiff(A, M1)
U2 = setdiff(omega, M2)
N2 = setdiff(A, M2)
isequal(U1, union(G, N1)) % prints 1 when U equals G union N
all(ismember(M1, M2))     % 1 when M(1) is a subset of M(2)
all(ismember(U2, U1))     % 1 when U(2) is a subset of U(1)
```

1.4 In Python

The file is `01_sets.py`. It prints the same sets as the hand calculation and confirms each test with `True`.

```
"""Sets and the Unseen model:  $U(t) = \Omega \setminus M(t)$ ."""
omega = set(range(1, 11))    # every state that actually occurs
A = set(range(1, 8))         # states creatures could in principle establish
G = omega - A                # absolute unseen
M = {1: {1, 2}, 2: {1, 2, 3, 5}} # what has been established by stage 1 and stage 2

for t in (1, 2):
    U = omega - M[t]          # unavailable at stage t
    N = A - M[t]              # relative unseen at stage t
    print("stage", t, "U =", sorted(U), "N =", sorted(N))
    print(" U equals G union N:", U == G | N)

print("M(1) is a subset of M(2):", M[1] <= M[2])
print("U(2) is a subset of U(1):", (omega - M[2]) <= (omega - M[1]))
```

What the calculation leaves open

The sets were chosen by us. The calculation confirms that the definitions fit together. It cannot tell you which real things belong in *A* and which in *G*. In the main book that boundary is drawn by revelation.

1.5 Read further

Any introduction to sets will cover this in its first chapter. Paul Halmos, *Naive Set Theory*, is short and classic. Daniel Velleman, *How to Prove It*, teaches sets and logic together for readers with no

background.

Logic and What Follows from What

2.1 The idea

Logic studies when a conclusion follows from what you already accept. Its most used sentence is “if S then B ”, written $S \Rightarrow B$. The sentence makes one promise: whenever S is true, B is true as well. It makes no promise about what happens when S is false.

The commonest mistake in everyday reasoning is to turn the sentence round. “If it rained, the street is wet” is true. “The street is wet, so it rained” does not follow, because a burst pipe also wets a street. The turned-round sentence, $B \Rightarrow S$, is called the *converse*.

Two more symbols appear in the book. $\neg A$ means “not A ”. $\forall x$ means “for every x ”. So $\forall x (P(x) \Rightarrow R(x))$ reads “for every x , if x has property P then x has property R ”.

Where the book uses this

Life-Threatening and Infectious Diseases writes $(S \Rightarrow B) \nRightarrow (B \Rightarrow S)$, the rain and wet street point in symbols. *Waswas, Unwanted Thoughts, and Religious Anxiety* writes $T \nRightarrow E$: having a thought does not mean endorsing it. *Sihr Against Prophet Muhammad* (صَلَّى اللهُ عَلَيْهِ وَعَلَى آلِهِ) uses $\forall$ to state two general claims precisely and to test whether they conflict. *What You Need Before Reading* uses $T \nvdash C$, which says that a set of assumptions T does not prove a claim C , and stresses that failing to prove C is different from proving that C is false.

2.2 By hand

A *truth table* lists every combination of true and false and works out the sentence in each. For “if S then B ” the rule is simple. The sentence is false in one case only, when S is true and B is false.

S	B	$S \Rightarrow B$	$B \Rightarrow S$
true	true	true	true
true	false	false	true
false	true	true	false
false	false	true	true

Look at the third row. $S \Rightarrow B$ is true there and $B \Rightarrow S$ is false. One row is enough to prove that the first sentence does not give you the second.

Now try “not proved” against “disproved” with a plain example. Your assumptions are “the shop opens at nine” and “it is now ten”. From these you cannot prove that the shop sells bread. You also cannot prove that it does not. Both questions stay open, and that is exactly what the pair of lines with $\nvdash$ in the book says.

2.3 In Octave

The file is `logic_table.m`. Octave writes true as 1 and false as 0. The expression $(\neg S) \mid B$ is how “if S then B ” is computed: it is true when S is false or B is true.

```
% Truth table for "if S then B" and its converse "if B then S".
disp('S B  S=>B  B=>S');
for S = [1 0]
    for B = [1 0]
        printf('%d %d    %d    %d\n', S, B, (!S) | B, (!B) | S);
    end
end
```

2.4 In Python

The file is `02_logic.py`.

```
"""Truth table for 'if S then B' and for its converse 'if B then S'."""
print("S      B      S=>B  B=>S")
for S in (True, False):
    for B in (True, False):
        s_implies_b = (not S) or B
        b_implies_s = (not B) or S
        print(f"{S!s:6} {B!s:6} {s_implies_b!s:6} {b_implies_s!s:6}")
```

What the calculation leaves open

Logic tests the step from assumptions to conclusion. It does not test the assumptions. A valid argument built on a false starting point still ends somewhere false.

2.5 Read further

Velleman’s *How to Prove It* again. For the limits of formal proof that the book mentions in connection with Gödel, Ernest Nagel and James Newman, *Gödel’s Proof*, is written for general readers.

Probability and Changing Your Mind

3.1 The idea

A probability is a number from 0 to 1 that says how strongly something is expected. Zero means it will not happen, one means it certainly will, and 0.5 is an even chance. Multiply by 100 and you have a percentage.

$P(H)$ is the probability of H . $P(H | E)$ is the probability of H *given* E , meaning the probability once you know E is true. Bayes' rule tells you how to get from one to the other:

$$P(H | E) = \frac{P(E | H) P(H)}{P(E)}.$$

In words: start from how likely the explanation was before, $P(H)$. Ask how likely the new information would be if the explanation were right, $P(E | H)$. Divide by how likely the information is overall, $P(E)$.

Where the book uses this

What You Need Before Reading introduces Bayes' rule, and then adds a second line, $B_{k+1} = B_k + \lambda_k (P(H | E) - B_k)$, to describe a person who moves only part of the way towards what the information supports. *How Beliefs Reinforce Themselves* builds on the same idea.

3.2 By hand

A screening test is the standard example, and it has nothing to do with the Unseen, which makes it a safe place to learn the rule.

Out of every 100 people, 2 have a certain condition, so $P(H) = 0.02$. The test is positive for 90 in 100 of those who have it, so $P(E | H) = 0.90$. It is also positive for 10 in 100 of those who do not. Someone tests positive. How likely is it that they have the condition?

Most people say about 90 per cent. Work it out with 1,000 people. Twenty have the condition, and 18 of them test positive. Of the 980 who do not have it, 98 test positive. So $18 + 98 = 116$ people test positive, and only 18 of them have the condition:

$$P(H | E) = \frac{18}{116} \approx 0.155.$$

About 16 per cent. The same sum with the formula is

$$P(E) = 0.90 \times 0.02 + 0.10 \times 0.98 = 0.116, \quad P(H | E) = \frac{0.90 \times 0.02}{0.116} \approx 0.155.$$

The answer is low because the condition is rare. That starting rarity, $P(H)$, is called the *base rate*, and forgetting it is one of the commonest errors in reasoning about any unusual explanation.

Now the second line. Suppose a person believed the explanation at 0.60 before the result, and each time they reconsider they move a quarter of the way towards 0.155, so $\lambda = 0.25$:

$$B_1 = 0.60 + 0.25 \times (0.155 - 0.60) = 0.489.$$

Repeat and you get 0.405, then 0.343, then 0.296. The person is heading the right way and is still far from 0.155 after four rounds. With $\lambda = 1$ they would arrive at once. With $\lambda = 0$ they would never move.

3.3 In Octave

The file is `bayes_update.m`.

```
% Bayes' rule with a base rate, then a partial belief update.
p_h = 0.02;           % P(H)
p_e_given_h = 0.90;   % P(E|H)
p_e_given_not_h = 0.10; % P(E|not H)

p_e = p_e_given_h * p_h + p_e_given_not_h * (1 - p_h)
p_h_given_e = p_e_given_h * p_h / p_e

belief = 0.60;
lam = 0.25;
for k = 1:4
    belief = belief + lam * (p_h_given_e - belief);
    printf('after step %d belief = %.4f\n', k, belief);
end
```

3.4 In Python

The file is `03_bayes.py`.

```
"""Bayes' rule with a base rate, then a partial belief update."""
p_h = 0.02           # P(H): 2 people in 100 have the condition
p_e_given_h = 0.90   # P(E|H): the test is positive for 90 in 100 who have it
p_e_given_not_h = 0.10 # P(E|not H): positive for 10 in 100 who do not

p_e = p_e_given_h * p_h + p_e_given_not_h * (1 - p_h)
p_h_given_e = p_e_given_h * p_h / p_e
print("P(E)  =", round(p_e, 4))
print("P(H|E) =", round(p_h_given_e, 4))

belief = 0.60        # what the person believed before
lam = 0.25           # how far they move towards the revised probability
for k in range(1, 5):
    belief = belief + lam * (p_h_given_e - belief)
    print("after step", k, "belief =", round(belief, 4))
```

What the calculation leaves open

The rule is only as good as the three numbers you feed it. For a medical test those numbers come from studies. For most claims about the Unseen nobody has such numbers, and the book says so. In those cases Bayes' rule is a way of thinking clearly about base rates and alternatives. It is not a machine for producing a percentage.

3.5 Read further

Joseph Blitzstein and Jessica Hwang, *Introduction to Probability*, has a free online edition and begins from nothing. Daniel Kahneman, *Thinking, Fast and Slow*, describes how people actually handle base rates. OpenStax, *Introductory Statistics*, is a free textbook at openstax.org.

Rules That Update Step by Step

4.1 The idea

Many of the book's formulas have the same shape: the next value is worked out from the present one. The subscript counts the steps. K_t is the value now and K_{t+1} is the value one step later. A rule of this kind is called a *recurrence*.

The plainest one in the book is $K_{t+1} = K_t \cup \{x_t\}$. It says that your knowledge after step t is your knowledge before, with one new item x_t added. If knowledge really grew like that, the order in which you learned things would not matter.

Where the book uses this

How Beliefs Reinforce Themselves sets that simple rule beside a more realistic one, $K_{t+1} = U(K_t, I(x_t, K_t))$. In the second rule the new item is first *interpreted* in the light of what you already hold, and only then absorbed. The chapter's point is that the two orders of learning can then end in different places.

4.2 By hand

Take a rule simple enough to do in your head. You hold a number. Each time a new number arrives, you move halfway towards it:

$$K_{t+1} = K_t + \frac{1}{2} (x_t - K_t).$$

Start at 5. Suppose you hear 0 and then 10. After the first you are at 2.5, and after the second at 6.25. Now hear them the other way round. After 10 you are at 7.5, and after 0 at 3.75.

The information was the same and the order was different, and you finish at 6.25 in one case and 3.75 in the other. Whatever you heard last counts for more. This is a toy, and it makes the one point the chapter needs: once an update depends on your present state, order matters.

4.3 In Octave

The file is `update_order.m`. The line beginning `update = @` defines a small function of two inputs.

```
% An update rule in which the order of the information matters.
update = @(k, x) k + (x - k) / 2;
start = 5;
zero_then_ten = update(update(start, 0), 10)
ten_then_zero = update(update(start, 10), 0)
```

4.4 In Python

The file is 04_updates.py.

```
"""An update rule in which the order of the information matters."""  
def update(k, x):  
    return k + (x - k) / 2      # move halfway towards the new item  
  
start = 5  
print("0 then 10:", update(update(start, 0), 10))  
print("10 then 0:", update(update(start, 10), 0))
```

What the calculation leaves open

The halfway rule was invented for the example. Real people do not update by a fixed fraction, and the book does not claim they do. The example demonstrates that order *can* matter under a simple rule. How much it matters for a real person is a question for psychology.

4.5 Read further

Recurrences are covered in any discrete mathematics text. For the psychology of how prior belief shapes what people take from new information, see Kahneman's book named in Part 3.

Lists of Answers, Differences, and Limits

5.1 The idea

Sometimes one number hides too much. To describe where a community stands on ten separate questions, you need ten entries. An ordered list of entries is called a *vector*, written in bold: $\mathbf{s} = (s_1, s_2, \dots, s_n)$.

Three small tools then appear.

The symbol $\sum$ means “add up”. $\sum_{i=1}^n$ adds a term for each i from 1 to n .

The symbol $\#[\dots]$ is a counter. It is 1 when the statement in brackets is true and 0 when it is false.

$\max\{a, b\}$ is the larger of two numbers and $\min\{a, b\}$ is the smaller.

Where the book uses this

Sectarian Difference Without Purity Scores records a community’s positions as $\mathbf{s}(C)$ and counts disagreements between two communities with $d(\mathbf{s}_A, \mathbf{s}_B) = \sum_i \#[s_i(A) \neq s_i(B)]$. It then compares the authority an adviser *asserts*, k_T , with the authority their expertise *supports*, a_T , using $e_T = \max\{0, k_T - a_T\}$ for the excess and $b_T = \min\{k_T, a_T\}$ for the supported part.

5.2 By hand

Write two lists of ten answers, using 1, 2, and 3 as labels for the possible positions:

Question	1	2	3	4	5	6	7	8	9	10
$\mathbf{s}_A$	1	2	1	3	2	1	1	2	3	1
$\mathbf{s}_B$	1	2	2	3	2	1	3	2	1	1

Go along the columns and put a mark wherever the two rows differ. They differ at questions 3, 7, and 9, so $d = 3$. That is all the formula does. The labels are names, so the count is the same whichever positions you call 1, 2, or 3.

Now the authority comparison, with the book’s own figures. Suppose an adviser’s expertise supports a level of 0.8 on some question. If they assert 0.4, the excess is $\max\{0, 0.4 - 0.8\} = \max\{0, -0.4\} = 0$. Holding back never produces a negative excess. If they assert 0.9, the excess is $\max\{0, 0.9 - 0.8\} = 0.1$ and the supported part is $\min\{0.9, 0.8\} = 0.8$.

For two advisers with the same figures, the book’s index adds the term $k_T k_U - b_T b_U$ for each question. At 0.9 asserted and 0.8 supported that is $0.9 \times 0.9 - 0.8 \times 0.8 = 0.81 - 0.64 = 0.17$, the value printed in the chapter.

5.3 In Octave

The file is `profiles.m`. The expression `s_a != s_b` compares the lists entry by entry and gives 1 wherever they differ.

```
% Counting disagreements, and asserted against supported authority.
s_a = [1 2 1 3 2 1 1 2 3 1];
s_b = [1 2 2 3 2 1 3 2 1 1];
disagreements = sum(s_a != s_b)

a = 0.8;                                % supported level, same for both advisers
for k = [0.4 0.9]                        % asserted level, same for both advisers
    e = max(0, k - a);
    b = min(k, a);
    term = k * k - b * b;
    printf('asserted %.1f excess %.2f term %.2f\n', k, e, term);
end
```

5.4 In Python

The file is `05_profiles.py`.

```
"""Counting disagreements, and asserted against supported authority."""
s_a = [1, 2, 1, 3, 2, 1, 1, 2, 3, 1]
s_b = [1, 2, 2, 3, 2, 1, 3, 2, 1, 1]
d = sum(1 for x, y in zip(s_a, s_b) if x != y)
print("disagreements:", d)

def excess(k, a):
    return max(0, k - a)

def supported(k, a):
    return min(k, a)

for k in (0.4, 0.9):
    # asserted level, same for both advisers
    a = 0.8
    # supported level, same for both advisers
    e, b = excess(k, a), supported(k, a)
    term = k * k - b * b
    print("asserted", k, "excess", round(e, 2), "term", round(term, 2))
```

What the calculation leaves open

A count of three disagreements says where two lists differ. It says nothing about which questions are weighty, which answers are well supported, or how any individual behaves. The levels 0.8 and 0.9 are illustrations. Nobody has measured an adviser's authority on a scale from zero to one.

5.5 Read further

Vectors and summation are introduced in OpenStax *College Algebra*. The count of differing entries is known as the Hamming distance, and any introduction to coding theory or discrete mathematics explains it.

Comparing Priorities as Shares

6.1 The idea

Two lists of weights are hard to compare if their totals differ. The cure is to turn each list into *shares*: divide every weight by the total of its own list, so that each list adds up to 1. This step is called *normalising*.

The vertical bars $|x|$ give the *absolute value*, the size of a number with its sign removed. $|-0.4| = 0.4$.

Where the book uses this

How Religious Priorities Can Shift compares the priorities the sources give to a chosen set of teachings, w^* , with the priorities a community's conduct suggests, w_c . It defines

$$D_O(c) = \frac{1}{2} \sum_{i=1}^n \left| \frac{w_c(o_i)}{\sum_j w_c(o_j)} - \frac{w^*(o_i)}{\sum_j w^*(o_j)} \right|.$$

The result is 0 when the two lists give the same shares and 1 when they have nothing in common.

6.2 By hand

Take three teachings. A model gives them source-based weights of 5, 3, and 2. Observation of what a community spends its effort on suggests 1, 3, and 6.

Both lists total 10, so the shares are easy: 0.5, 0.3, 0.2 for the sources and 0.1, 0.3, 0.6 for conduct. Subtract entry by entry: $-0.4, 0, 0.4$. Remove the signs: 0.4, 0, 0.4. Add: 0.8. Halve: $D = 0.4$.

Why halve? Whatever one teaching gains in share, the others together must lose. Every shift is therefore counted twice, once as a gain and once as a loss, and the half removes the double counting. Reading the answer: four tenths of the community's attention sits somewhere other than where the model's source-based weights would put it.

6.3 In Octave

The file is `priorities.m`.

```
% Comparing two lists of priorities as shares: D_O(c).
w_star = [5 3 2];
w_c = [1 3 6];
share_star = w_star / sum(w_star)
share_c = w_c / sum(w_c)
D = 0.5 * sum(abs(share_c - share_star))
```

6.4 In Python

The file is 06_priorities.py.

```
"""Comparing two lists of priorities as shares: D_0(c)."""
w_star = [5, 3, 2]    # source-based priorities in the model
w_c = [1, 3, 6]       # estimated actual priorities

share_star = [w / sum(w_star) for w in w_star]
share_c = [w / sum(w_c) for w in w_c]
D = 0.5 * sum(abs(c - s) for c, s in zip(share_c, share_star))
print("shares (sources):", share_star)
print("shares (actual): ", share_c)
print("D =", round(D, 3))
```

What the calculation leaves open

The arithmetic is the easy part. Someone has to justify the weights 5, 3, and 2, and revelation does not hand out a numerical scale for every duty. Someone also has to estimate conduct fairly. The formula compares two lists that people supplied. It discovers nothing about a religion.

6.5 Read further

This measure is known in statistics as the total variation distance between two distributions. Blitzstein and Hwang's book covers the background.

Sharing Credit Fairly

7.1 The idea

Several people bring about a result together. How much did each one add? Lloyd Shapley proposed an answer in 1953. Imagine the people arriving one at a time. Each time someone arrives, note how much the group’s achievement rises. Do this for every possible order of arrival and average what each person added. That average is the person’s *Shapley value*.

You need one ingredient: a rule $v(S)$ that says what each possible group S could achieve by itself. Groups are called *coalitions*.

The exclamation mark in the formula is a *factorial*. $3! = 3 \times 2 \times 1 = 6$ is the number of orders in which three people can arrive.

Where the book uses this

What We Can and Cannot Control uses the Shapley value to explain why a person’s calculated share of an outcome depends on who else the model includes. It writes the share as $\varphi_i(v)$ and, for a model of scope F , as $C_i(x, F)$. The chapter keeps that calculated share apart from moral responsibility and from the Stoic idea of what is within your control.

7.2 By hand

Begin with the book’s two cases. Two qualified people, one task, and either can do it alone. If you arrive first, the group goes from 0 to 1, so you add 1. If you arrive second, the task was already possible, so you add 0. Your average is $\frac{1}{2}$. In the second case neither can do it alone. Arriving first you add 0, arriving second you add 1, and the average is again $\frac{1}{2}$.

Now three people. The task needs person 1 together with at least one helper, and helpers 2 and 3 are interchangeable. So $v = 1$ for the groups $\{1, 2\}$, $\{1, 3\}$, and $\{1, 2, 3\}$, and $v = 0$ for every other group. List the six orders and write what each arrival adds.

Order of arrival	Person 1 adds	Person 2 adds	Person 3 adds
1, 2, 3	0	1	0
1, 3, 2	0	0	1
2, 1, 3	1	0	0
2, 3, 1	1	0	0
3, 1, 2	1	0	0
3, 2, 1	1	0	0
Average	4/6	1/6	1/6

Person 1 receives two thirds and each helper one sixth. The shares add to 1, which is v of the whole group. That always happens, and the book writes it as $\sum_i \varphi_i(v) = v(N) - v(\emptyset)$.

Try the book's central observation yourself. Remove person 3 from the model and recalculate. With only persons 1 and 2, each needs the other, so each receives one half. Person 1 did nothing different, and their share fell from two thirds to one half because the model now holds fewer people.

7.3 In Octave

The file is `shapley_three.m`. `perms` lists every order.

```
% Shapley values by averaging over every joining order.
% Three people. The task needs person 1 and at least one helper.
v = @(S) double(any(S == 1) && numel(S) >= 2);

players = [1 2 3];
orders = perms(players);
totals = zeros(1, 3);
for r = 1:rows(orders)
    group = [];
    for p = orders(r, :)
        totals(p) = totals(p) + v([group p]) - v(group);
        group = [group p];
    end
end
shares = totals / rows(orders)
sum(shares)
```

7.4 In Python

The file is `07_shapley.py`. It runs the two-person cases first and then the three-person case.

```
"""Shapley values by averaging over every joining order."""
from itertools import permutations

def shapley(players, v):
    totals = {p: 0.0 for p in players}
    orders = list(permutations(players))
    for order in orders:
        group = frozenset()
        for p in order:
            totals[p] += v(group | {p}) - v(group)
            group = group | {p}
    return {p: totals[p] / len(orders) for p in players}

either = lambda S: 1 if len(S) >= 1 else 0          # either person can do it alone
both = lambda S: 1 if len(S) == 2 else 0           # they need each other
print("substitutes:", shapley(["a", "b"], either))
print("need both: ", shapley(["a", "b"], both))

# Three people. The task needs person 1 and at least one helper.
def v(S):
    return 1 if 1 in S and len(S) >= 2 else 0
result = shapley([1, 2, 3], v)
```

```
print("three people:", {p: round(x, 4) for p, x in result.items()})  
print("sum of shares:", round(sum(result.values()), 4))
```

What the calculation leaves open

A Shapley value divides whatever the rule v describes, among whoever the model includes. Change either and the shares change. It is a share of a modelled outcome. It is not a verdict on blame, merit, or what a person could have controlled.

7.5 Read further

Martin Osborne, *An Introduction to Game Theory*, covers cooperative games at an introductory level. The original paper is Lloyd Shapley, "A Value for n -Person Games" (1953). The author's preprint on this subject is listed in the main book under *The Author's Published Preprints*.

Choices That Depend on Other People's Choices

8.1 The idea

Game theory also studies situations where each side chooses separately and the result depends on both choices. Each outcome is given a *payoff*, a number saying how much a side prefers it. A pair of choices is a *Nash equilibrium* when neither side could do better by changing its own choice while the other side stays put.

Where the book uses this

Iblis, Temptation, and Human Conflict uses the best-known example, the Prisoner's Dilemma, to explain how two groups that would both gain from cooperation can end in mutual harm because each fears being exploited.

8.2 By hand

Here is the book's table. Each cell gives the first group's payoff and then the second group's.

	Second cooperates	Second defects
First cooperates	3, 3	0, 5
First defects	5, 0	1, 1

Put yourself in the first group and take the columns one at a time. If the second group cooperates, you get 3 by cooperating and 5 by defecting, so defecting pays more. If the second group defects, you get 0 by cooperating and 1 by defecting, so defecting pays more again. Whatever they do, your own payoff is higher if you defect. The second group reasons the same way.

Both defect and both receive 1. Had both cooperated, both would have received 3. Add the pairs: the totals are 6, 5, 5, and 2, and the outcome that separate self-interest produces is the worst of the four.

8.3 In Octave

The file is `dilemma.m`.

```
% Best replies in the two-group dilemma used in the book.
% Rows are the first group's choice, columns the second group's.
% Index 1 means cooperate, index 2 means defect.
first = [3 0; 5 1];    % payoffs to the first group
second = [3 5; 0 1];   % payoffs to the second group
```

```
names = {'C', 'D'};
for other = 1:2
    [~, best] = max(first(:, other));
    printf('if the other group plays %s the best reply is %s\n', names{other}, names{best});
end
totals = first + second
```

8.4 In Python

The file is 08_dilemma.py.

```
"""Best replies in the two-group dilemma used in the book."""
payoff = {("C", "C"): (3, 3), ("C", "D"): (0, 5),
          ("D", "C"): (5, 0), ("D", "D"): (1, 1)}

for other in ("C", "D"):
    best = max(("C", "D"), key=lambda mine: payoff[(mine, other)][0])
    print("if the other group plays", other, "the best reply is", best)

for cell, (p1, p2) in payoff.items():
    print(cell, "total", p1 + p2)
```

What the calculation leaves open

The numbers 0, 1, 3, and 5 only rank the outcomes. Many real conflicts have a different structure, and people do cooperate when they can build trust, meet repeatedly, or hold to a moral commitment. The table describes one trap. It does not say that every conflict is this trap.

8.5 Read further

Osborne's book begins with exactly this game. Robert Axelrod, *The Evolution of Cooperation*, is a readable study of how cooperation survives when the game is repeated.

Systems, Feedback, and Chaos

9.1 The idea

A *system* here means anything whose condition changes over time. Its condition at step k is its *state*, x_k . The book writes a general rule of change as

$$x_{k+1} = F(x_k, u_k, w_k).$$

Read it as a sentence. The next state depends on the present state x_k , on what you deliberately do, u_k , and on outside disturbances you did not choose, w_k . A second line, $y_k = H(x_k) + \varepsilon_k$, says that what you observe, y_k , is a view of the state with some error ε_k mixed in.

Feedback means that the system's present condition is used to decide what to do next. A heater with a thermostat is the usual example: it compares the room with a target and heats more when the gap is large.

A system is *linear* when doubling the input doubles the response and two inputs together give the sum of their separate responses. The book writes this as $L(ax_1 + bx_2) = aL(x_1) + bL(x_2)$. Everything else is *nonlinear*. Some nonlinear systems are *chaotic*: they follow a fixed rule exactly, and yet two starts that differ by a hair drift far apart.

Where the book uses this

The Fundamental Sciences introduces the state and observation equations. *Chaos, Nonlinear Systems, and Questions About Sorcery's Reach* explains linearity, thresholds, and the growth of small differences, $\|\delta x(t)\| \approx \|\delta x(0)\|e^{\lambda t}$. *Diseases, Illnesses, and Competing Explanations* writes a person's condition as a state influenced by medical, psychological, social, and spiritual inputs. *When Circumstances Affect a Ruling* uses a thermostat to explain the difference between correcting practice and quietly moving the standard. *Direction, Stability, and the Path Through Time* uses stability as a limited analogy.

9.2 By hand

9.2.1 Feedback

A room is at 10 degrees and the target is 20. Each step, the heater closes 30 per cent of the remaining gap:

$$x_{k+1} = x_k + 0.3(20 - x_k).$$

The gap is 10, so the first step adds 3 and the room is at 13. The gap is now 7, so the next step adds 2.1 and the room is at 15.1. Then 16.57, 17.60, 18.32, 18.82. The room approaches 20 and the steps get smaller because the correction is proportional to the gap.

Now the book's warning. Suppose that instead of heating the room, someone resets the target to 10. The display reports a gap of $10 - 10 = 0$. Nothing was corrected. The standard was moved to where the room already was. In control-engineering terms the book calls this *setpoint capture*.

9.2.2 Chaos

Take the rule $x_{k+1} = 4x_k(1 - x_k)$ and start at $x_0 = 0.2$. One step gives $4 \times 0.2 \times 0.8 = 0.64$. The next gives $4 \times 0.64 \times 0.36 = 0.9216$, then 0.2890, and so on. The rule is fixed and involves no chance at all.

Run it again from 0.2000001, a start that differs in the seventh decimal place. For a while the two runs agree closely. The gap between them roughly doubles at each step, and doubling is relentless: after 20 steps the two runs differ by about 0.1, and after 25 they are unrelated.

Compare this with the book's formula. For this rule the growth rate is $\lambda = \ln 2 \approx 0.693$, and $e^{0.693} = 2$, so the gap doubles each step. Starting from 10^{-7} , twenty doublings give $10^{-7} \times 2^{20} \approx 0.105$, which matches what the runs print. The number $e \approx 2.718$ is the base of natural growth, and $e^{\lambda t}$ is the standard way to write "multiplied by the same factor in every equal interval".

9.3 In Octave

The files are `feedback.m` and `chaos.m`.

```
% A room heater with feedback, and what "moving the standard" does.
goal = 20;
x = 10;
for k = 1:6
    x = x + 0.3 * (goal - x);
    printf('step %d temperature %.2f gap %.2f\n', k, x, goal - x);
end
cold_room = 10;
captured_goal = cold_room;
reported_gap = captured_goal - cold_room
```

```
% Two nearly identical starts under the rule x -> 4x(1-x).
x = 0.2;
y = 0.2000001;
for k = 0:30
    if mod(k, 5) == 0
        printf('step %2d x = %.6f y = %.6f difference = %.2e\n', k, x, y, abs(x - y));
    end
    x = 4 * x * (1 - x);
    y = 4 * y * (1 - y);
end
growth_per_step = exp(0.693)
```

9.4 In Python

The files are `09_feedback.py` and `10_chaos.py`.

```
"""A room heater with feedback, and what 'moving the standard' does."""
```

```

goal = 20.0
x = 10.0
for k in range(1, 7):
    error = goal - x
    x = x + 0.3 * error          # correct 30 per cent of the gap each step
    print("step", k, "temperature", round(x, 2), "gap", round(goal - x, 2))

# Moving the standard: redefine the goal as whatever the room already is.
cold_room = 10.0
captured_goal = cold_room
print("reported gap after moving the standard:", captured_goal - cold_room)

```

```

"""Two nearly identical starts under the rule  $x \rightarrow 4x(1-x)$ ."""
import math
x, y = 0.2, 0.2000001
for k in range(0, 31):
    if k % 5 == 0:
        print("step", k, "x =", round(x, 6), "y =", round(y, 6),
              "difference =", f"{abs(x - y):.2e}")
        x, y = 4 * x * (1 - x), 4 * y * (1 - y)

print("growth factor per step,  $e^{0.693}$  =", round(math.exp(0.693), 3))
print("1e-7 doubled 20 times =", 1e-7 * 2 ** 20)

```

What the calculation leaves open

Chaos is a statement about prediction. It tells you that small uncertainty at the start can grow until a forecast is worthless. It does not let anyone trace a later event back to a chosen small cause, and it gives nobody a way to steer a distant outcome. A state equation for a person is a way of organising questions. Nobody has measured its terms.

9.5 Read further

Steven Strogatz, *Nonlinear Dynamics and Chaos*, is the standard introduction and is unusually readable. James Gleick, *Chaos*, tells the story without equations. Karl Åström and Richard Murray, *Feedback Systems*, is a full introduction to control that the authors make available free online.

Waves, Frequency, and the Spectrum

10.1 The idea

A wave is a repeating disturbance that travels. Its *frequency* f is how many times it repeats each second, measured in hertz (Hz). Its *wavelength* λ is the distance from one crest to the next. The two are tied together by the speed v at which the wave travels:

$$v = f\lambda.$$

Light, radio, infrared, microwaves, and X-rays are all *electromagnetic* waves. They differ only in frequency, and in empty space they all travel at the speed of light, about 300,000 kilometres per second. Sound is a different kind of wave. It is a vibration of air or another material, it cannot cross empty space, and in air it travels at about 343 metres per second.

The prefixes matter. One terahertz (THz) is a million million hertz, 10^{12} . One nanometre (nm) is a thousand-millionth of a metre, 10^{-9} .

Where the book uses this

A Precise Definition of ‘The Unseen’ notes that unaided human vision covers roughly 400 to 790 THz, a narrow band of what instruments can detect. *The Fundamental Sciences* separates electromagnetic waves from sound when it examines proposals about hidden signals and recitation. *What Human Superiority Means* and *How Biology, Experience, and Choice Shape a Person* explain why the spectrum gives no place to jinn or to spiritual perception. *Physical Analogies and Sympathetic Magic* tests claims about fields and resonance.

10.2 By hand

Convert the edges of human vision from frequency to wavelength. Rearranged, the formula is $\lambda = v/f$. For 400 THz,

$$\lambda = \frac{3.00 \times 10^8}{400 \times 10^{12}} = 7.5 \times 10^{-7} \text{ m} = 750 \text{ nm},$$

which is deep red. For 790 THz the same division gives about 380 nm, which is violet. On a calculator, enter 3×10^8 , divide by 790×10^{12} , and multiply by 10^9 to read the answer in nanometres.

Now a sound. The note A above middle C is 440 Hz, so its wavelength in air is $343/440 \approx 0.78$ metres, about the length of your arm. Compare the two results. Visible light has a wavelength smaller than a bacterium and a musical note has one you could measure with a ruler. They are different physical things, and that is why the book will not let a claim about one be supported by a fact about the other.

10.3 In Octave

The file is `waves.m`. The operator `./` divides entry by entry.

```
% Frequency and wavelength: speed = frequency x wavelength.
c = 299792458;                % metres per second
f_thz = [400 790];
wavelength_nm = c ./ (f_thz * 1e12) * 1e9
sound_wavelength_m = 343 / 440
```

10.4 In Python

The file is `11_waves.py`. It uses the exact speed of light, so it prints 749.5 and 379.5 where the hand calculation rounded to 750 and 380.

```
"""Frequency and wavelength: speed = frequency x wavelength."""
c = 299_792_458                # speed of light in metres per second
for f_thz in (400, 790):
    wavelength_nm = c / (f_thz * 1e12) * 1e9
    print(f_thz, "THz ->", round(wavelength_nm, 1), "nm")

speed_of_sound = 343           # metres per second in air at about 20 degrees C
print("440 Hz sound ->", round(speed_of_sound / 440, 3), "m")
```

What the calculation leaves open

The spectrum catalogues physical radiation. The fact that human eyes detect only part of it proves that there is more than we see. It does not tell us what else there is, and it assigns no frequency to anything in the Unseen.

10.5 Read further

OpenStax *University Physics* and OpenStax *Astronomy* are free and cover waves and light from the beginning. Richard Feynman, *QED: The Strange Theory of Light and Matter*, explains light for a general audience.

The Physics the Book Refers To

This part has no calculations. It gives you a plain explanation of four pieces of physics that the main book mentions, enough to follow the argument, with a pointer to a fuller treatment.

11.1 The early universe

Light from distant galaxies is stretched towards the red, and the further the galaxy, the greater the stretch. The standard reading is that space itself has been expanding. Run the expansion backwards and the universe was once extremely hot and dense. Left over from that time is a faint glow of microwaves from every direction, the *cosmic microwave background*, at about 2.7 kelvin, which is 2.7 degrees above the coldest possible temperature.

From such measurements, cosmologists estimate that ordinary matter makes up about 5 per cent of the universe's energy content, *dark matter* about 27 per cent, and *dark energy* about 68 per cent. The last two are names for effects that are measured through gravity and expansion and whose nature is not yet known.

Where the book uses this

The Beginning and the Limits of Physics sets out what cosmology establishes and marks where questions of creation and purpose need another kind of answer. *A Precise Definition of 'The Unseen'* uses dark matter as an example of learning about something from its effects.

Read further: Steven Weinberg, *The First Three Minutes*. OpenStax *Astronomy* for a free textbook.

11.2 Quantum entanglement

Two particles that have interacted can be left in a joint condition. Measure one, and the result is linked to what a measurement on the other will give, even when they are far apart. John Bell proved in 1964 how to test whether such links could be explained by properties the particles carried all along, and experiments have since ruled that explanation out.

Two limits matter for the book. The link appears only when the two sets of results are brought together and compared. And it cannot be used to send a message or to make anything happen at the other end.

Where the book uses this

Physical Analogies and Sympathetic Magic examines whether entanglement can support the idea that things once in contact stay connected, and explains why shared history or shared DNA does not create it. *When Rejected Ideas Later Enter Science* uses the history of Bell's work as a case of a strange idea becoming testable.

Read further: the entry on Bell's theorem in the *Stanford Encyclopedia of Philosophy*, listed in Part 14.

11.3 The exclusion principle and surfaces

Wolfgang Pauli's exclusion principle says that two electrons cannot occupy the same quantum state. One consequence is that matter is solid: atoms resist being pushed into one another. The atoms at the surface of a material are also in a different situation from those inside, for a simpler reason: they have neighbours on one side only. Surfaces are where a material meets and exchanges with its surroundings.

Where the book uses this

Surfaces, Boundaries, and Physical Analogies builds on this to discuss boundaries that allow both exchange and exposure, in materials, cells, computers, and the senses, and then states why a boundary does not establish a hidden cause.

Read further: Richard Feynman, *The Character of Physical Law*.

11.4 Thermodynamics, entropy, and thresholds

Thermodynamics is the physics of heat and energy. *Entropy* measures how many microscopic arrangements are consistent with what you observe on the large scale. Left alone, systems move towards arrangements that are more numerous, which is why heat spreads out and does not gather itself.

A *phase transition* is a sudden change of kind, such as water freezing, that happens when a gradual change crosses a *threshold*.

Where the book uses this

The Fundamental Sciences uses thermodynamic comparisons, and *Chaos, Nonlinear Systems, and Questions About Sorcery's Reach* and *Physical Analogies and Sympathetic Magic* use thresholds to explain how a small push can produce a large change when a system is already close to its limit.

Read further: OpenStax *University Physics*, Volume 2.

Chains, Trees, and Networks

12.1 The idea

Draw a dot for each person and an arrow from each person to those they learned from. The picture is a *graph*. The dots are *nodes* and the arrows are *edges*. A graph with no loops in which everything traces back to one origin is a *tree*. A single unbranched line of nodes is a *chain*.

Where the book uses this

Hadith, The Living Record of the Prophet's (صَلَّى اللهُ عَلَيْهِ وَعَلَى آلِهِ وَسَلَّمَ) *Life* describes how a report reaches a collector through a chain of named transmitters, the *isnad*, and how scholars assess each link. *The Tree of Transmission* widens the picture from single chains to the network through which teaching travelled.

12.2 By hand

A collector heard a report from two students. The first student heard it from one teacher. The second heard it from that same teacher and from another teacher. Both teachers heard it from the source.

Draw it, then trace every route from the collector back to the source with your finger. There are three:

1. collector, first student, teacher, source;
2. collector, second student, teacher, source;
3. collector, second student, other teacher, source.

Look at what the routes share. Routes 1 and 2 pass through the same teacher. If that teacher made an error, both routes carry it, so they are not two independent confirmations. Route 3 goes through a different teacher and adds something the first two do not. Counting routes is easy. Seeing which routes share a link is the real work.

12.3 In Python

The file is `12_chains.py`. A network like this is most naturally written as a table of who heard from whom, which Python calls a dictionary. There is no Octave version of this example, because Octave is built for numbers more than for named relationships.

```
"""A small transmission network: who heard from whom, and how many routes."""
heard_from = {
    # each person lists their teachers
    "collector": ["student1", "student2"],
    "student1": ["teacher"],
    "student2": ["teacher", "other_teacher"],
    "teacher": ["source"],
```

```
"other_teacher": ["source"],
"source": [],
}

def routes(person):
    if person == "source":
        return [["source"]]
    found = []
    for t in heard_from[person]:
        for r in routes(t):
            found.append([person] + r)
    return found

for r in routes("collector"):
    print(" <- ".join(r))
print("number of routes:", len(routes("collector")))
```

What the calculation leaves open

A graph records who is linked to whom. It does not record whether a transmitter was reliable, whether two people really met, or whether the wording was preserved. Those judgements belong to Hadith scholarship, and the picture is only a way of laying its findings out.

12.4 Read further

Graphs are introduced in any discrete mathematics text. For the scholarly discipline itself, begin with the sources the main book cites in its Hadith chapter.

A Philosophy Toolkit

The main book leans on a handful of ideas from philosophy. None needs a calculation. Each is given here in a paragraph, with the chapter that uses it and a place to read more.

13.1 Kinds of reasoning

Deduction moves from general statements to what must follow. If all the premises are true and the form is valid, the conclusion is certain. *Induction* moves from many observed cases to a general expectation, and its conclusions are probable, never certain. *Abduction*, or inference to the best explanation, starts from an observation and asks which of the available explanations fits it best. Doctors, detectives, and historians reason this way most of the time.

Where the book uses this

What You Need Before Reading sets out these kinds of reasoning. *Keeping Different Kinds of Knowledge Separate* asks three questions of any claim: what do the religious sources support, what does observation support, and does the conclusion follow?

13.2 Knowledge and its sources

Epistemology is the study of knowledge: what it is to know something and what justifies a belief. People come to hold beliefs through perception, reasoning, memory, and *testimony*, the word philosophers use for learning from what others report. Most of what anyone knows rests on testimony, which is why the reliability of a source and of the route by which its report reached you matters so much.

13.3 Testing a claim

Karl Popper argued that a scientific claim must be *falsifiable*. There has to be some possible observation that would count against it. A claim that is compatible with every possible outcome tells you nothing about which outcome to expect.

Where the book uses this

When Rejected Ideas Later Enter Science separates claims that have not yet been measured from claims defined so that no measurement could bear on them.

13.4 Correlation and cause

Two things can occur together without one causing the other. A third factor may cause both, the order may be the reverse of what you assumed, or the pairing may be coincidence. Proving cause

needs more: a comparison with what would have happened otherwise, and a mechanism that could carry the effect. A related point is the *nocebo effect*, in which expecting harm produces or worsens real symptoms.

Where the book uses this

The opening chapter, *The Subject of This Book*, makes this point with a ritual followed by an illness. It returns in *Diseases, Illnesses, and Competing Explanations* and in the chapters on possession.

13.5 Analogy

An analogy says that two things are alike in some respect. It is a tool for explaining and for suggesting questions. It proves nothing by itself, because two things that are alike in one respect can differ in every other. When the book compares a spiritual matter with a physical system, it marks the comparison [B] and states where the likeness stops.

13.6 Models

A *model* is a deliberately simplified description. A map is a model of a city. It is useful because it leaves things out, and it misleads you the moment you forget that it did.

13.7 The Stoic division

The Stoic philosopher Epictetus opened his handbook by dividing things into those that are up to us, such as our judgements and choices, and those that are not, such as our bodies, reputations, and what other people do. Peace of mind, he taught, comes from putting effort into the first and accepting the second.

Where the book uses this

What We Can and Cannot Control starts from this division and then explains why questions of contribution and collective duty need more than a two-way split.

13.8 The limits of formal proof

In 1931 Kurt Gödel proved that any consistent formal system rich enough for ordinary arithmetic contains true statements that the system cannot prove. The result is about formal systems of a specific kind. It is often stretched into claims about the limits of all human knowledge, which it does not support.

Where the book uses this

What You Need Before Reading refers to Gödel's theorems when it discusses what a formal proof can and cannot settle.

13.9 Read further

The *Stanford Encyclopedia of Philosophy* (plato.stanford.edu) and the *Internet Encyclopedia of Philosophy* (iep.utm.edu) are free, written by specialists, and have entries on every idea in this part. Epictetus, *Enchiridion*, is a few pages long. Karl Popper, *The Logic of Scientific Discovery*. Judea Pearl and Dana Mackenzie, *The Book of Why*, on cause and correlation. Nagel and Newman, *Gödel's Proof*.

PART 14

Finding Your Way

14.1 Which part goes with which chapter

Chapter of the main book	Parts of this booklet
A Precise Definition of ‘The Unseen’	1, 10, 11
Sihr Against Prophet Muhammad ﷺ	2
What You Need Before Reading	2, 3, 13
Keeping Different Kinds of Knowledge Separate	2, 13
How Beliefs Reinforce Themselves	3, 4
The Tree of Transmission	12
The Beginning and the Limits of Physics	1, 11
Surfaces, Boundaries, and Physical Analogies	11
Direction, Stability, and the Path Through Time	9
What We Can and Cannot Control	7, 13
What Human Superiority Means	5, 10
How Biology, Experience, and Choice Shape a Person	10, 9
Physical Analogies and Sympathetic Magic	10, 11
When Rejected Ideas Later Enter Science	1, 11, 13
Hadith, The Living Record of the Prophet’s ﷺ	12
(عليه وعلى آله وسلم) Life	
How Religious Priorities Can Shift	6
When Circumstances Affect a Ruling	9
Sectarian Difference Without Purity Scores	5
The Fundamental Sciences	9, 10, 11
Chaos, Nonlinear Systems, and Questions About Sorcery’s Reach	9, 11
Diseases, Illnesses, and Competing Explanations	2, 9, 13
Life-Threatening and Infectious Diseases	2
Iblis, Temptation, and Human Conflict	8
Waswas, Unwanted Thoughts, and Religious Anxiety	2

14.2 Symbols used in the main book

Symbol	Read it as	Part
$\in$	is a member of	1
$\subseteq$	is a subset of	1
$\cup$	union, everything in either	1
$\setminus$	set difference, the first without the second	1
$\emptyset$	the empty set	7
$\Rightarrow$	if ... then	2
$\nRightarrow$	does not imply	2
$\neg$	not	2
$\forall$	for every	2
$\vdash, \nvdash$	proves, does not prove	2
$P(H \mid E)$	probability of H given E	3
x_k, x_{k+1}	the value now, the value one step later	4
$\sum$	add up	5
$\#[\dots]$	1 if the statement is true, otherwise 0	5
max, min	the larger, the smaller	5
$ x $	size of x without its sign; for a set, how many members	6
$n!$	factorial, $n \times (n-1) \times \dots \times 1$	7
$\varphi_i(v)$	the Shapley value of participant i	7
$e^{\lambda t}$	steady multiplication at rate λ over time t	9
$\ \delta x\ $	the size of a small difference	9
$\approx$	is approximately	10
$\equiv_P$	alike in every property listed in P	below

The last symbol comes from *Practices That Look Similar but Are Not the Same*. $a \equiv_P b$ says that a and b give the same answer for every property in a chosen list P . Two practices can match on every property in a short list, such as posture and wording, and part company as soon as you add intention or the object of worship to the list. Whether two things count as “the same” always depends on which properties you agreed to compare.

14.3 Where to learn more

These sources are free, and each starts from the beginning.

- Khan Academy, khanacademy.org, for arithmetic, algebra, probability, and physics in short lessons with practice.
- OpenStax, openstax.org, for complete textbooks on algebra, statistics, physics, and astronomy.
- MIT OpenCourseWare, ocw.mit.edu, for full university courses with lecture notes.
- 3Blue1Brown, 3blue1brown.com, for visual explanations of mathematical ideas.
- The *Stanford Encyclopedia of Philosophy*, plato.stanford.edu.
- GNU Octave documentation, octave.org, and the Python tutorial, docs.python.org.

For the history of mathematics and science in Muslim civilisation, which the main book treats in its appendix *The Forgotten Foundation*, Jim Al-Khalili, *Pathfinders: The Golden Age of Arabic Science*, is a good general introduction.